Higher

# VECTORS

## DANIEL FRIEDRICH

Senior lecturer, University of Edinburgh

**SUBJECTS STUDIED AT SCHOOL**

Maths • Chemistry • Physics

**FURTHER EDUCATION:** MSc and PhD in Applied Mathematics

## CAREER JOURNEY SO FAR

Three years as a post-doc at the University of Edinburgh developing simulation software

Eight years as a lecturer in engineering mathematics at the University of Edinburgh

## FUTURE ASPIRATIONS

Mathematical models are becoming more and more important for the solution of societal challenges. It is my ambition to develop innovative teaching and research which motivates my students and bridges the gap between applications and advanced mathematics.

## Q&A WITH DANIEL

**What does your company/organisation do?**

The University of Edinburgh has two main roles:

1. Teach the next generation of graduates so that they can make a positive difference locally and globally.
2. Research and develop innovative solutions for today's and tomorrow's challenges.

**What types of activities do you do in your job?**

My work as a lecturer in the School of Engineering is almost equally split between teaching undergraduate and postgraduate students and research in mathematical methods to decarbonise energy systems.

**What does a typical day at work look like for you?**

My work as a lecturer in the School of Engineering is almost equally split between teaching undergraduate and postgraduate students and research in mathematical methods to decarbonise energy systems.

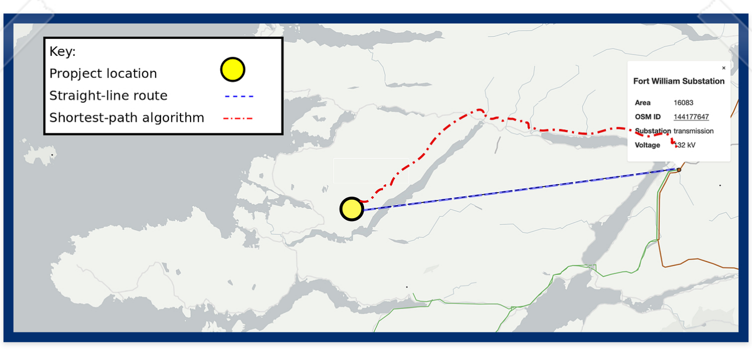
**What are your favourite things about your job?**

Both the teaching as well as the research parts of my job can be frustrating but also very satisfying. It is great when the students grasp a difficult concept or when we are solving a tricky research question.

## HOW DANIEL USES VECTORS AT WORK

I teach vector calculus to engineering students because it is needed for many engineering applications such as to calculate the length of tortuous routes or the mass of arbitrary shaped objects.

I use vectors in my research to find optimal routes to connect wind farms to the electricity grid and to calculate the power output of the wind farm for different wind speeds and directions.




## ACTIVITIES

Problem 1 - Power output of a wind turbine

The power output of a wind turbine depends on the speed and direction of the wind relative to the wind turbine. Usually the wind turbine is pointed in the same direction as the wind.

Unfortunately, the wind turbine has a fault: it can't follow the wind direction and is stuck in direction t = (4, 3). Calculate the speed and angle of the wind relative to the wind turbine for a wind velocity of w = (3.4).

Problem 2 - Non- horizontal wind

A similar reduction in output power would occur if the wind is not blowing horizontally.

The wind turbine has now been fixed and can follow the wind again but only in the x-y plane. However, now the wind is blowing up the slope the wind turbine has been built on. Calculate the speed and angle of the wind relative to the wind turbine for a wind velocity of w = (3, 4, 1)

# Solutions

## Problem 1

To find the speed of the wind in the direction of the wind turbine, we have to calculate the scalar product of the wind **w** with the unit vector of the wind turbine direction **t/|t|.** To calculate the unit vector we need to find the magnitude of the wind turbine direction

$$|\boldsymbol{t}| = \sqrt{\mathbf{4^2 + 3^2}} = \sqrt{\mathbf{16 + 9}} = \sqrt{\mathbf{25}} = \mathbf{5}$$

This can now be used to calculate the scalar product

$$w \cdot\ t/|t| = \frac{1}{5}\sum_{i=1}^{2}(w_i t_i) = \frac{1}{5}(w_1 t_1 + w_2 t_2) = \frac{1}{5}(3 * 4 + 4 * 3) = \frac{12+12}{5} = 4.8$$

which gives the speed of the wind in the direction of the wind turbine.

The angle can be calculated from the following formula

$$w \cdot t = |w||t| \cos\theta$$

which can be rearranged to

$$\theta = \cos^{-1}\left(\frac{w \cdot t}{|w||t|}\right) = \cos^{-1}\left(\frac{24}{5 * 5}\right) = \cos^{-1}(0.96) = 0.28\, rad$$

## Problem 2

To find the speed of the wind in the direction of the wind turbine, we have to calculate the scalar product of the wind **w** with the unit vector of the wind turbine direction **t/|t|.** To calculate the unit vector we need to find the magnitude of the wind turbine direction which is **t** = (3, 4, 0).

$$|\boldsymbol{t}| = \sqrt{\mathbf{3^2 + 4^2 + 0^2}} = \sqrt{\mathbf{9 + 16}} = \sqrt{\mathbf{25}} = \mathbf{5}$$

This can now be used to calculate the scalar product

$$w \cdot\ t/|t| = \frac{1}{5}\sum_{i=1}^{3}(w_i t_i) = \frac{1}{5}(w_1 t_1 + w_2 t_2 + w_3 t_3) = \frac{1}{5}(3 * 3 + 4 * 4 + 1 * 0) = \frac{9+16+0}{5}$$

$$= 5$$

which gives the speed of the wind in the direction of the wind turbine.

The angle can be calculated from the following formula

$$w \cdot t = |w||t| \cos\theta$$

which can be rearranged to

$$\theta = \cos^{-1}\left(\frac{w \cdot t}{|w||t|}\right) = \cos^{-1}\left(\frac{25}{\sqrt{26} * 5}\right) = \cos^{-1}(0.98) = 0.20\, rad$$